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Some problems on (very)high age mortality dynamic modeling

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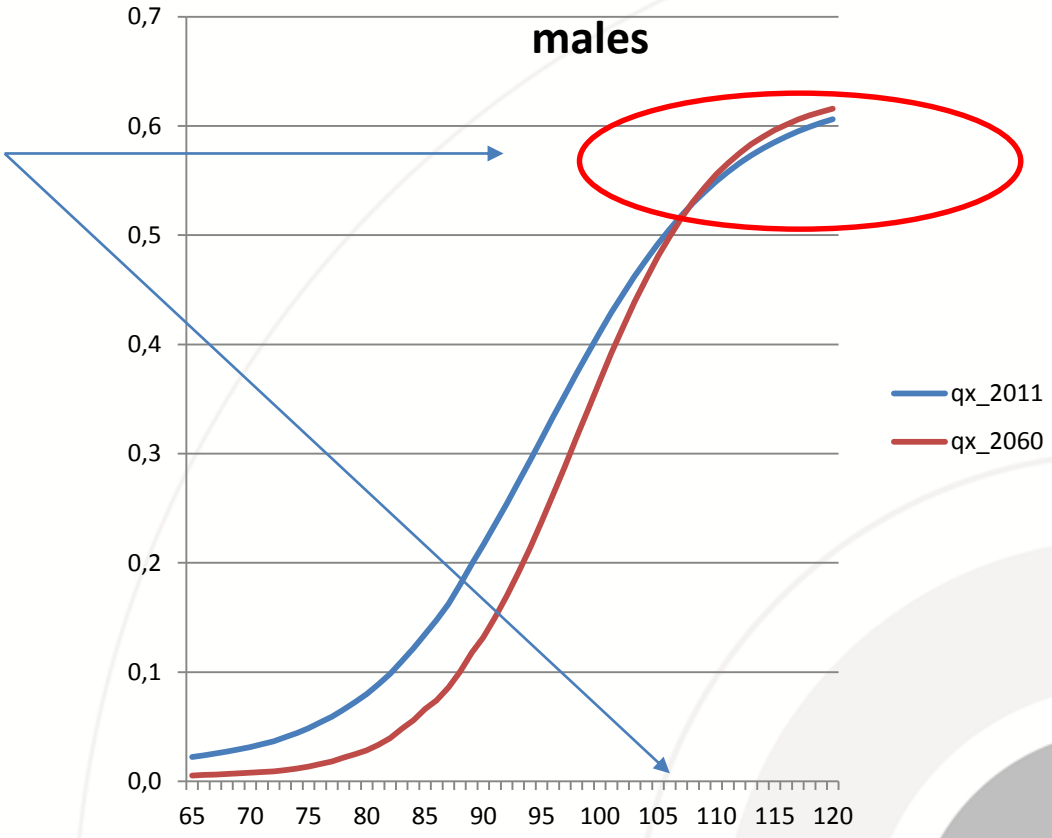
Motivation

- Once upon a time…
- …in “a prediction” some probabilities of death were increasing in time.

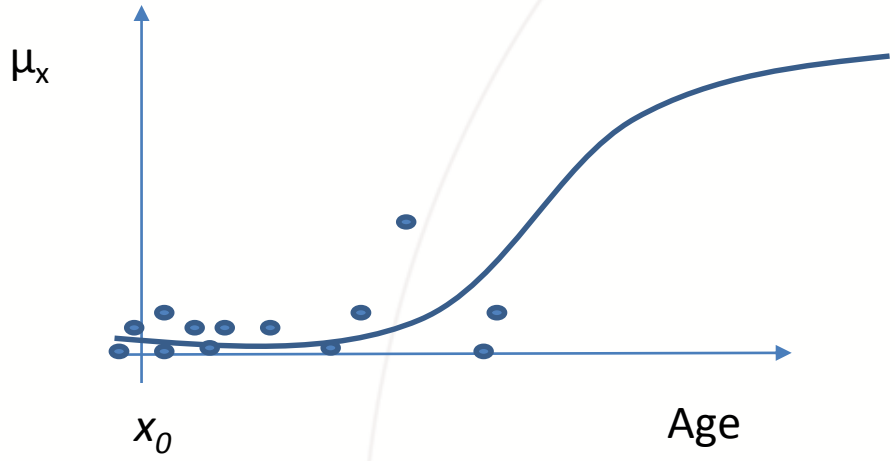
- How did this happen???

- Error in data?
- Error in estimates?
- SW bug?

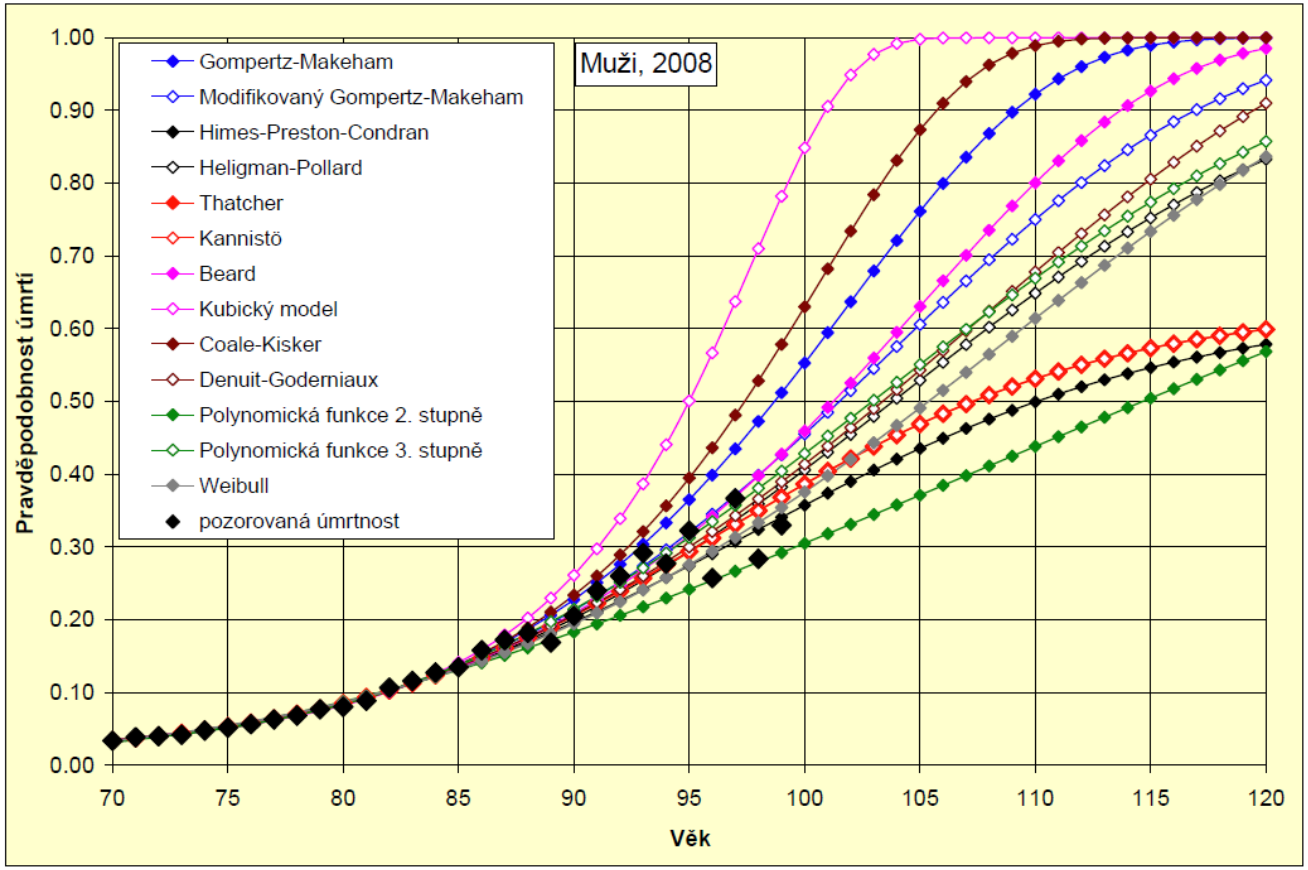


(Very) High Age Models

- (Very) high age data is very scarce
- For some ages there are no observations…yet…
- Mortality models are usually based on extrapolation with a parametric curve.



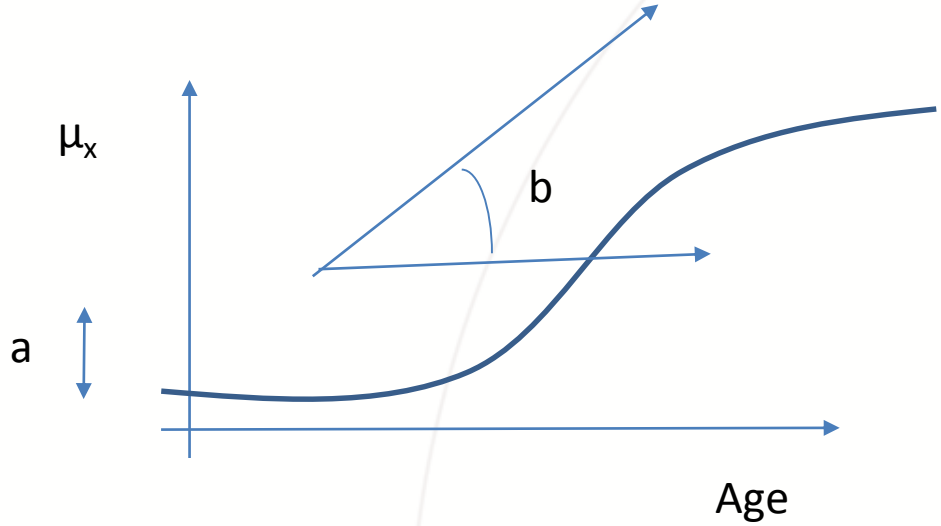
(Very) High Age Models



(Šídlo, Burcín, Tesárková 2010)

(Very) High Age Models- parameters

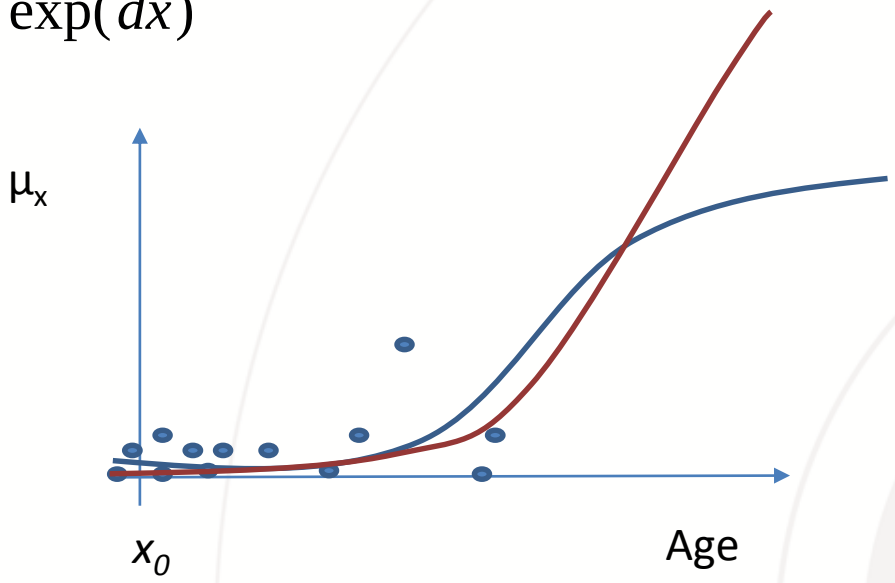
- Often based on two “main” parameters.
- Analogy to “Intersect” and “slope”.
- Related to inception and speed of increase.



(Very) High Age Models - examples

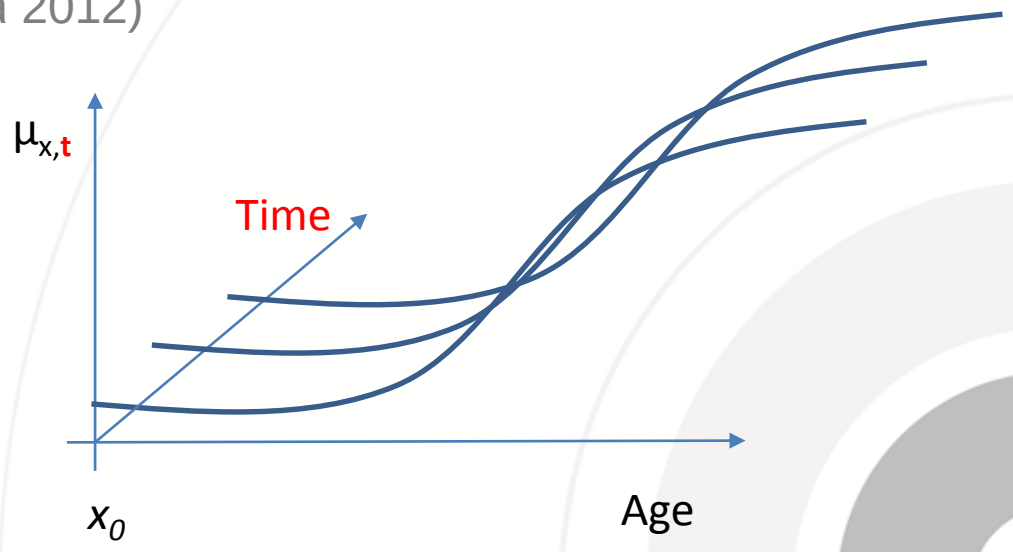
- Kannistö $$\mu_x = \frac{\exp(a+bx)}{1+\exp(a+bx)}$$

- Gompertz $$\mu_x = c\exp(dx)$$



Time dimension

- There is an evidence that parameters are not stable in time.
- => a_t , b_t
- Parallel shifts (Bongaarts 2005)
  - a_t evolves in time
- Age dependent shifts (Hulíková 2012)
  - a_t , b_t evolve in time

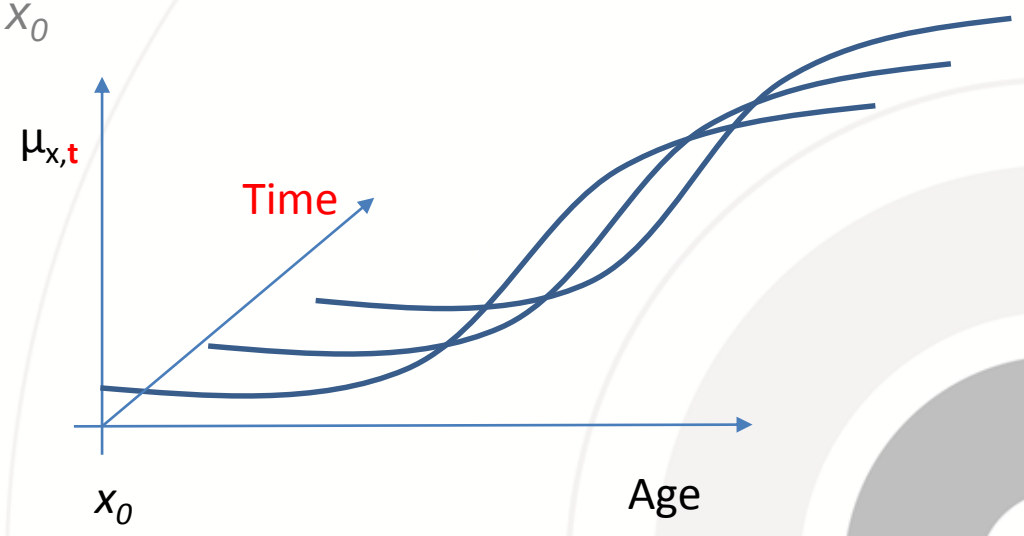


Rotation

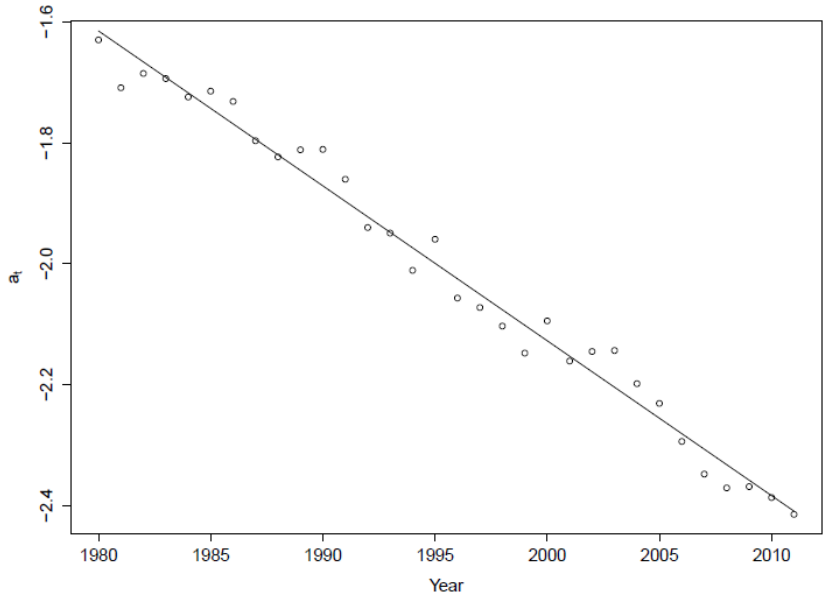
- Age dependent shifts
- a_t represents the level at x_0 – well observed usually **decreasing in time**

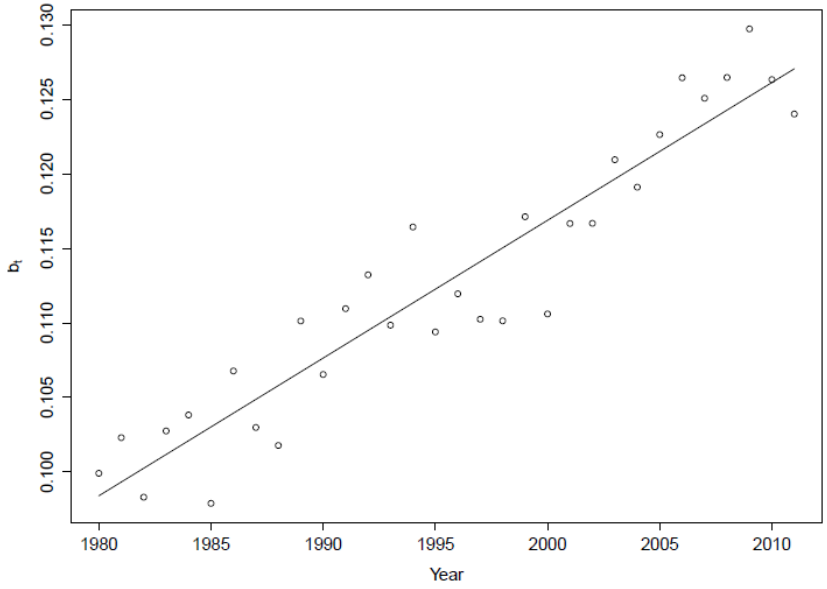
- The level at higher ages (100+):
  - Is not well observed
  - There are doubts if decreasing at all
  - Certainly **not as fast** as at x_0

- =>The **"slope" parameter** is **increasing** in time
- => **"Rotation"**

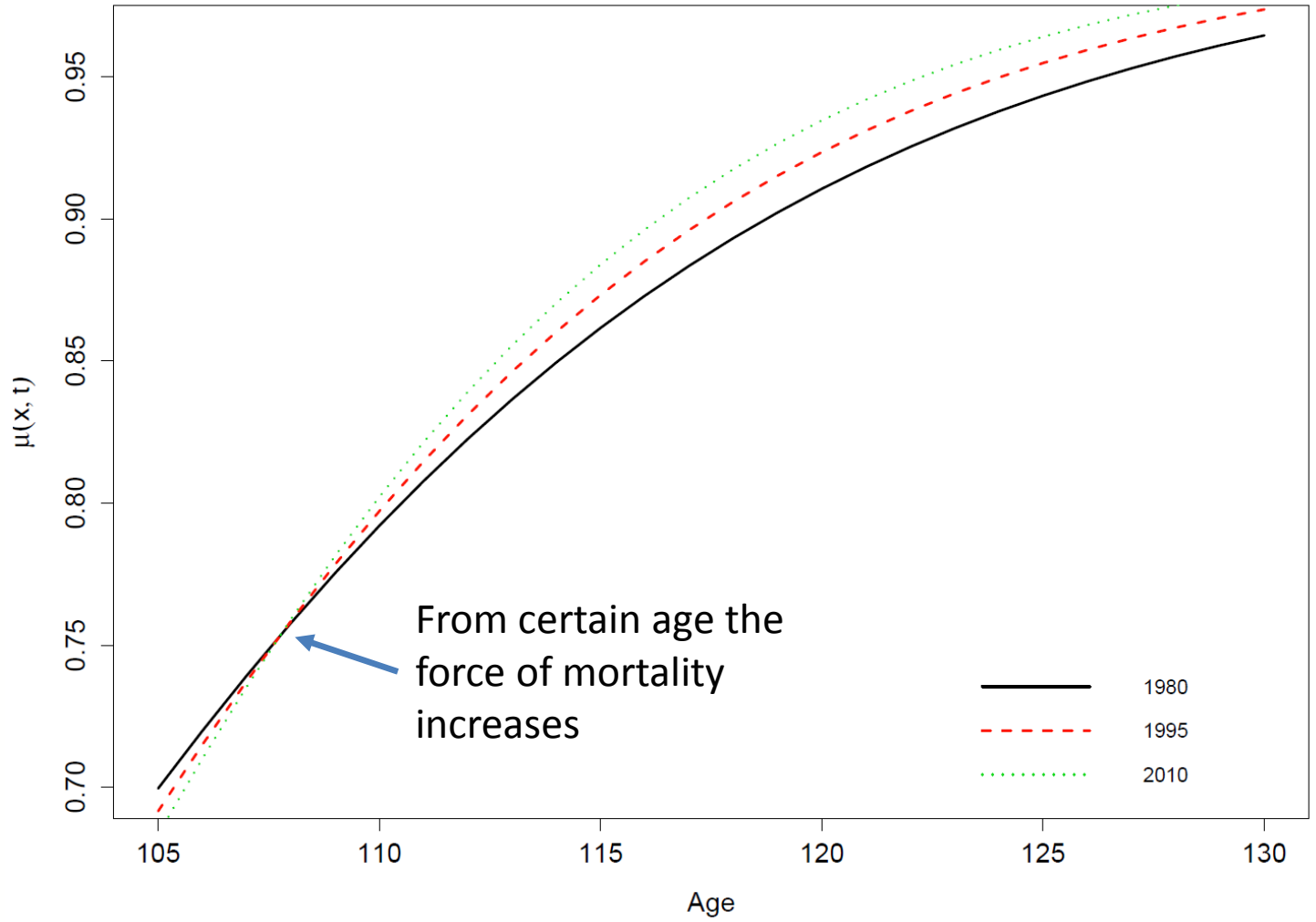


Parameters in time

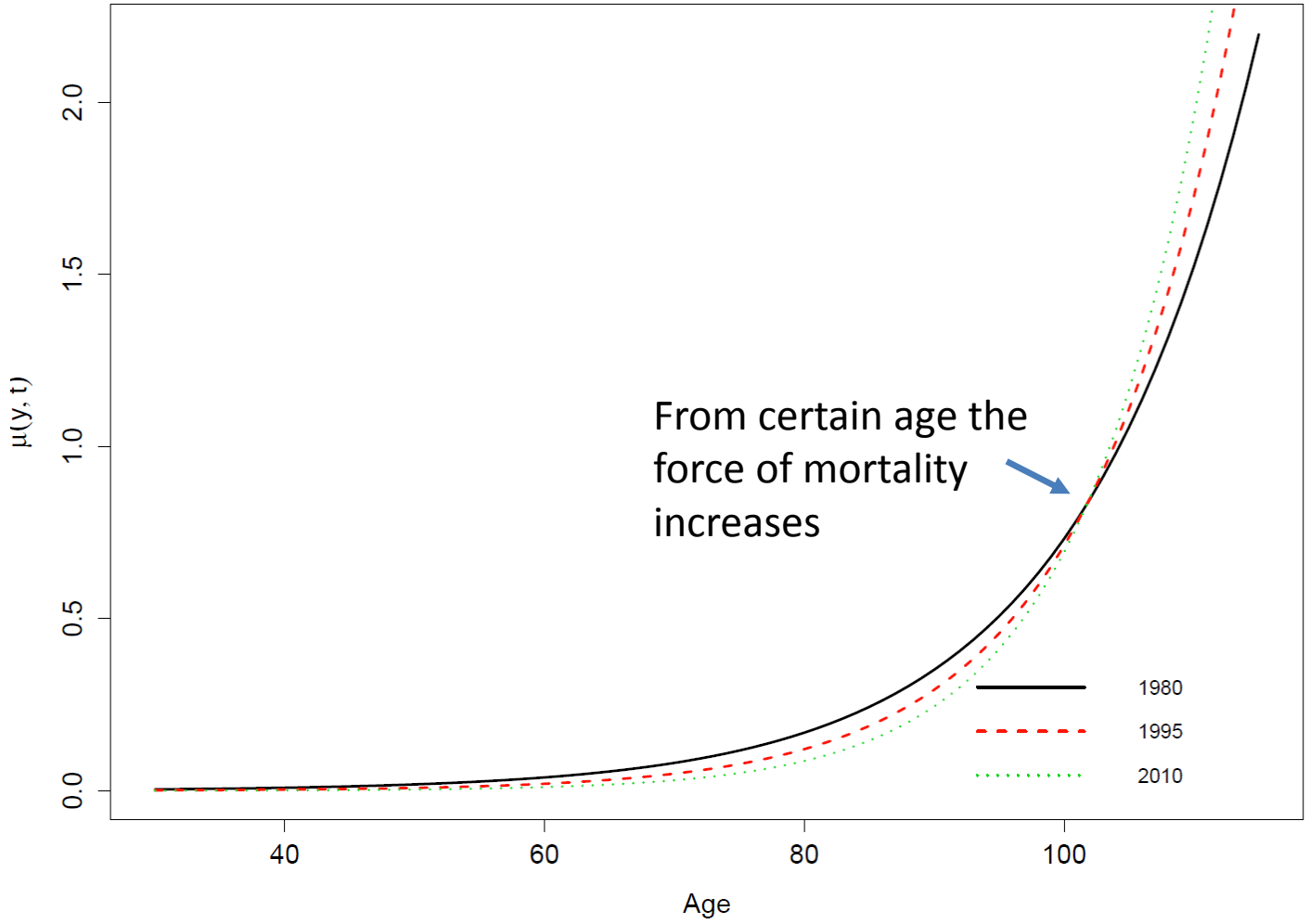




Rotation - Kannisto



Rotation - Gompertz



When does it start to increase?

- From certain age the force of mortality increases.
- Can be found analytically:
- Kannisto:

$$\frac{\partial \mu_{x,t}}{\partial t} > 0 \Rightarrow x\frac{db_t}{dt} > -\frac{da_t}{dt}$$

"trends" of the parameters

- Gompertz:

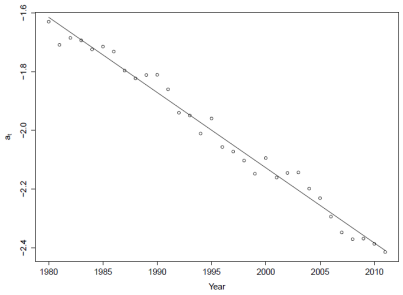
$$\frac{\partial \mu_{x,t}}{\partial t} > 0 \Rightarrow x\frac{dd_t}{dt} > -\frac{dc_t}{dt}$$

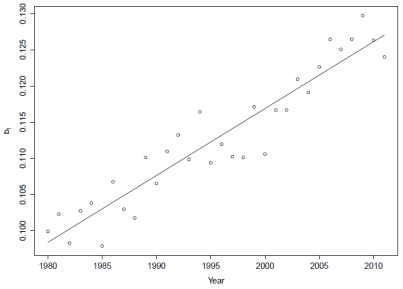
When does it start to increase?

$$\frac{\partial \mu_{x,t}}{\partial t} > 0 \Rightarrow x\frac{db_t}{dt} > -\frac{da_t}{dt}$$

"trends" of the parameters
(Constant if linear trend assumed)

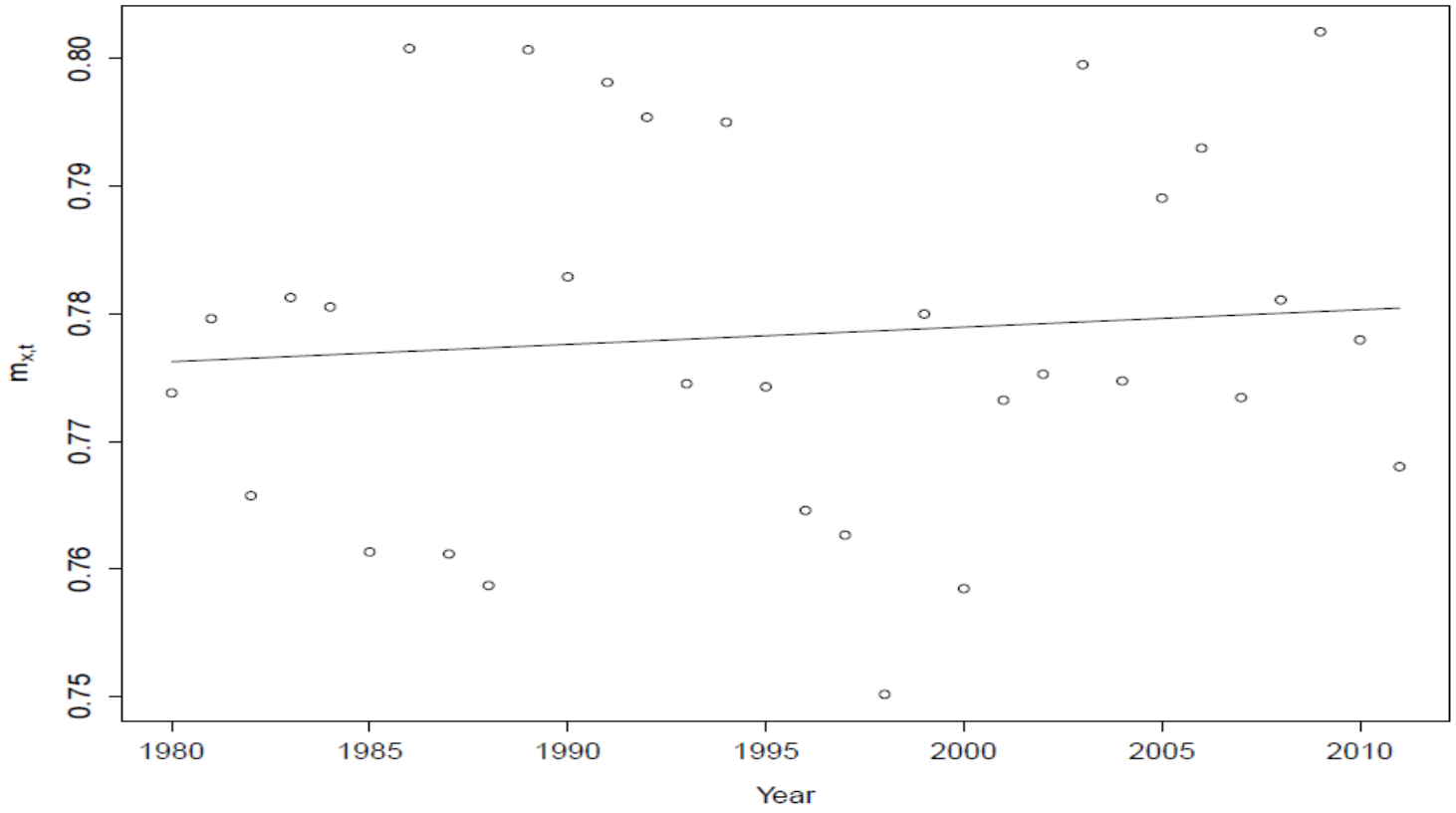
If a_t is decreasing and b_t is increasing in time, there will **always be** certain age from which the force of mortality increases in time.





For Czech males and Kannisto model, the threshold is 108 years

Central death rate HMDB Czech males 109 years



Conclusions

- If a_t is decreasing and b_t is increasing in time, there will always be certain age from which the force of mortality increases in time.
- This is purely due to model specification
- This is not observed…
- …and hard to consider realistic…

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Questions?

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References

- Bongaarts, John. "Long-range trends in adult mortality: Models and projection methods." *Demography* 42.1 (2005): 23-49.
- ŠÍDLO L., BURCIN B., TESÁRKOVÁ K. "NEJPOUŽÍVANĚJŠÍ METODY VYROVNÁVÁNÍ A EXTRAPOLACE KŘIVKY ÚMRTNOSTI A JEJICH APLIKACE NA ČESKOU POPULACI1)." *revue pro výkum populačního vývoje* 52 (2010): 77-89.
- Hulíková Tesarkova K. „Selected methods of mortality analysis focused on adults and the old-age. PhD thesis, Dept. Demography, Charles Univ., Prague, CZE, 2012.